



UNIVERSITY OF CALGARY
HASKAYNE SCHOOL OF BUSINESS

Corporate Finance

Fisher Separation Theorem

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Which shareholder shall I serve?

Intertemporal choice of consumption

- No wealth allocation across periods
- With productive investment opportunities
- With capital market but without productive investment opportunities
- With both productive investment opportunities and capital market

Fisher Separation Theorem: implications and critique

Chapter 4 of the textbook

Fisher Separation Theorem (1930)

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Irving Fisher is possibly the greatest American economist ever, but he also said just nine days before the 1929 market crash that the stock prices have reached 'a permanently high plateau'.

- Always beware: predicting the future like an oracle is a risky endeavor...

His seminal work, '*The Theory of Interest: As Determined by Impatience to Spend Income and Opportunity to Invest It*', was published in 1930.

- Fisher convincingly presents an equilibrium model of the economy, that allows, notably, to justify the maximization of present value as the goal of the firm.

This seminal work also introduces what is now referred to as the **Fisher Separation Theorem**,

- 'justifying the delegation of production decisions to firms that maximize present value, without any direct dependence on shareholder preferences, and justifying the separation of firm financing and production decisions' (Rubinstein, 2006).

Which shareholder shall I serve? (but shall I care?)

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Empirical evidence suggests that individuals like to **smooth-out consumption**. But the degree to which individuals desire and are able to smooth-out consumption vary between individuals.

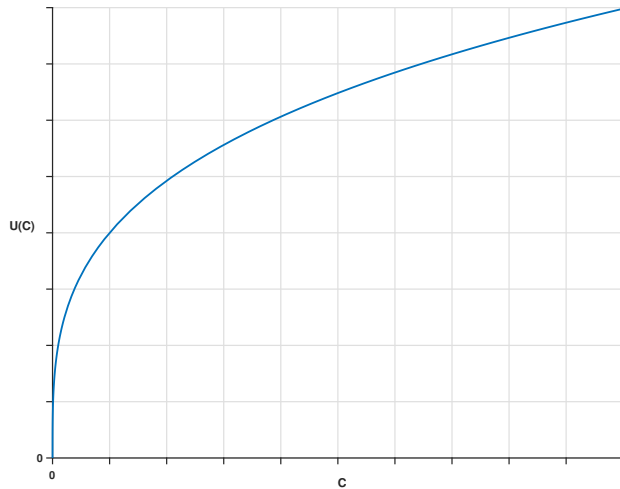
From the perspective of Irving Fisher, there is a tug of war between the 'Impatience to Spend Income' and the 'Opportunity to Invest It'.

A firm having a large number of shareholders, its management seemingly has to decide which financial policy best address the financial needs of its shareholders, but clearly shareholders all have different 'time preferences' (impatience versus patience).

As part of a broader modeling exercise under certain assumptions, Fisher suggests that, when using the right criterion (the net present value, aka NPV), the firm as well as each shareholder would make the same investments (i.e. pursue the same set of projects), making the individual differences between shareholders irrelevant. Management must maximize the wealth of shareholders by maximizing the enterprise value of the firm by making well-advised decisions using the appropriate finance know-how.

Utility of Consumption

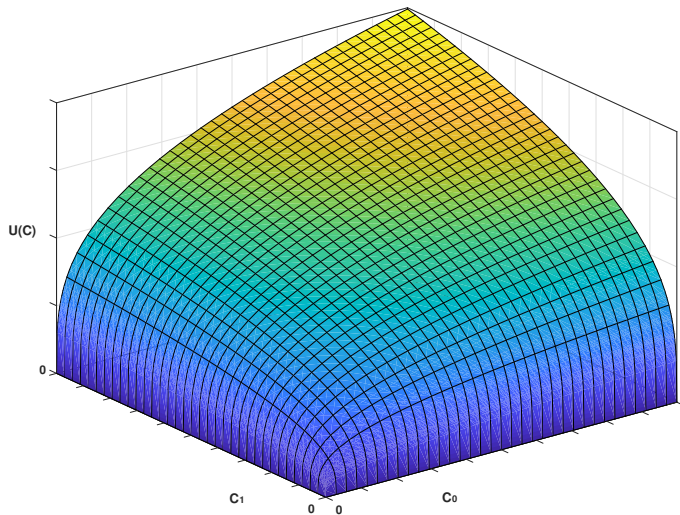
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As consumption C increases, utility $U(C)$ increases (prefer more to less), but at a declining rate (as $U = C^{0.3}$).

Two-period joint utility function (a surface)

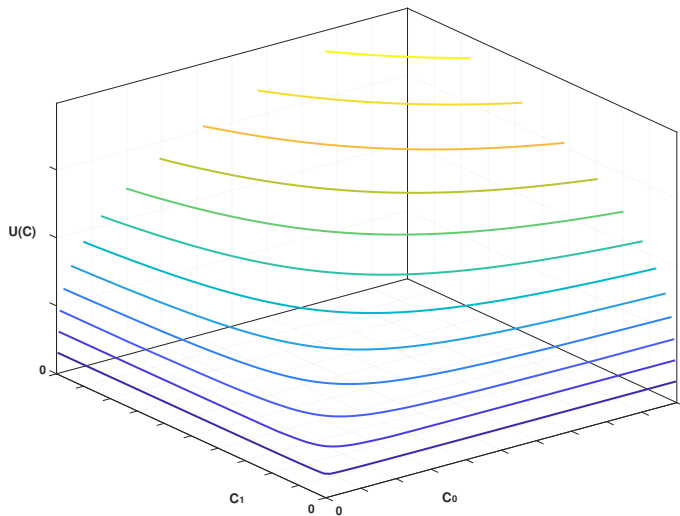
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C_0 : consumption today; C_1 : consumption tomorrow, $U(C)$ is the joint utility of C_0 and C_1

Two-period utility function: indifference curves

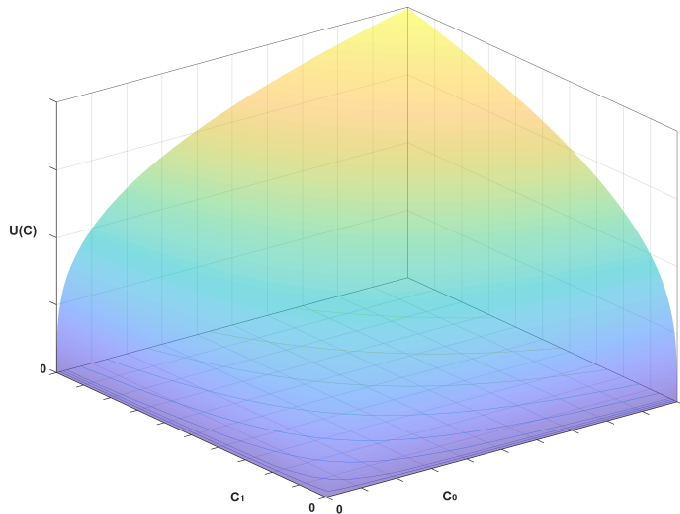
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Indifference curve: same utility for various combinations of C_0 and C_1 .

Two periods utility function: indifference curves

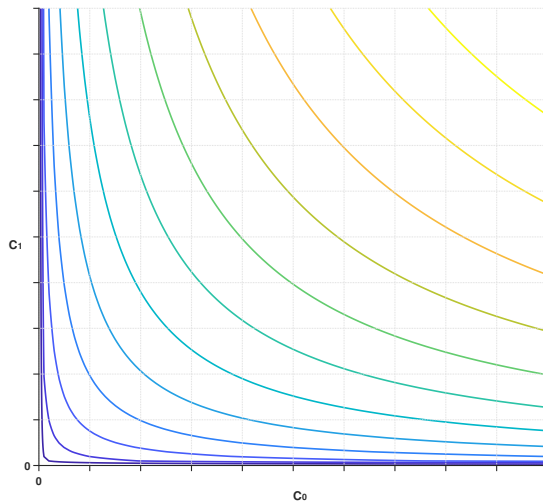
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Project the indifference curves on the c_0 c_1 plane.

Utility indifference curves per C_0 versus C_1

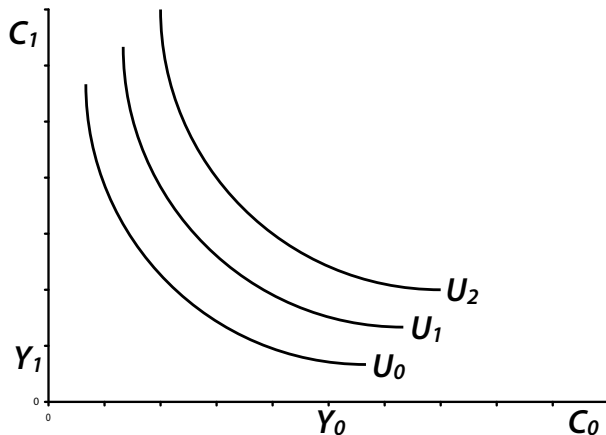
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The indifference curves increase in utility in the direction of the north-east quadrant.

Utility for the inter-temporal consumption choice

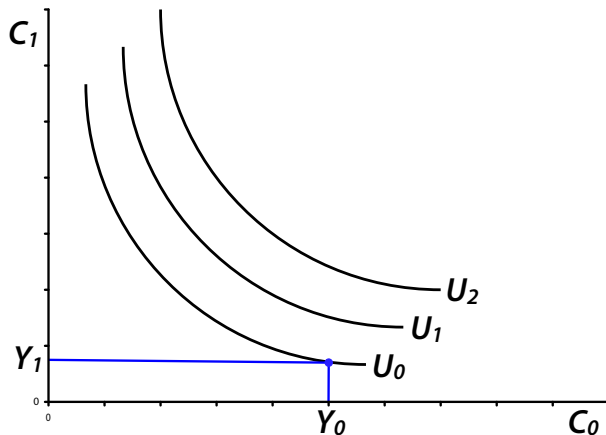
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$U_0 < U_1 < U_2$ (more is better); U is convex to origin; Y is the period-specific income.

What if no wealth allocation across periods?

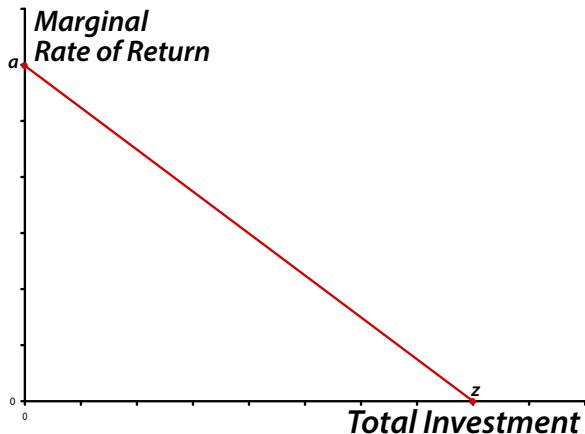
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The absence of a mechanism to allocate (transfer) wealth across periods precludes consumption smoothing ($C_0 = Y_0 \gg C_1 = Y_1$ is not good, since feast to famine is very much disliked by humans).

What if productive investment opportunities exist?

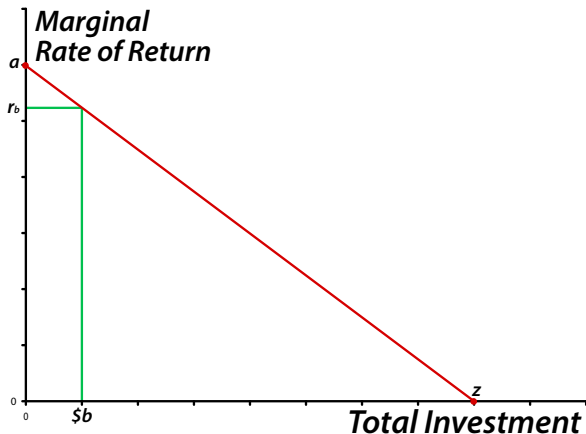
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Assumption: investments are independent of one another and perfectly divisible
As usual, choose first the investment with the highest rate of return and so forth (a to b to ... to z).

What if productive investment opportunities exist?

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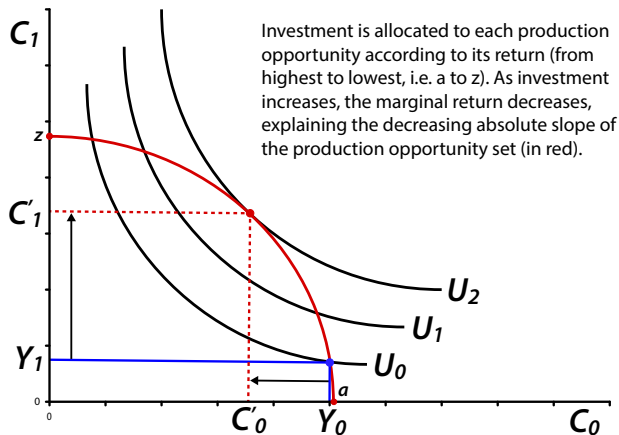


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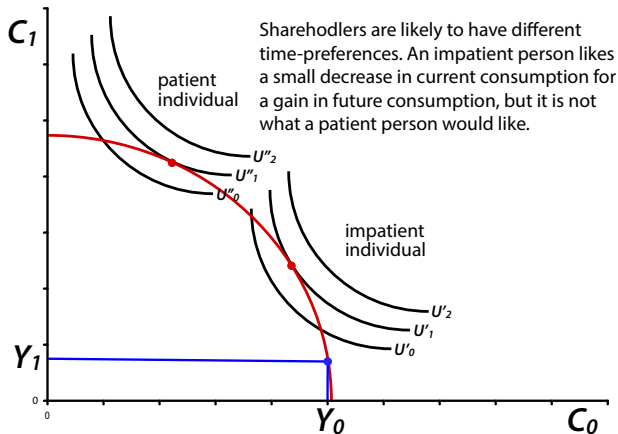
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$I_0 = Y_0 - C'_0$ (investment = income minus consumption, i.e. forego consumption of I_0 today) $\rightarrow C'_1 \gg C_1 = Y_1$
 The opportunity to invest today allows for more consumption tomorrow along the production opportunity set (in red from a to z), while increasing utility substantially from U_0 to U_2 : excellent!

But which production opportunity set is optimal?

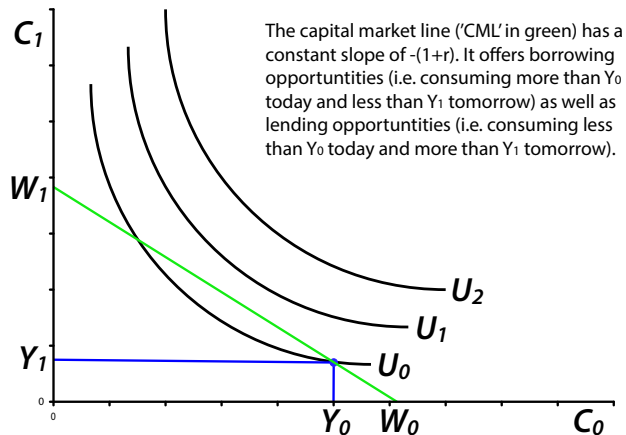
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As $Y_1 \ll Y_0$, foregoing current consumption in exchange for much improved future consumption makes most shareholders better off, but management is unable to choose an investment opportunity set that is optimal to most shareholders (problematic as some shareholders will be significantly 'happier' than most shareholders).

What if capital market? (but no production opportunities)

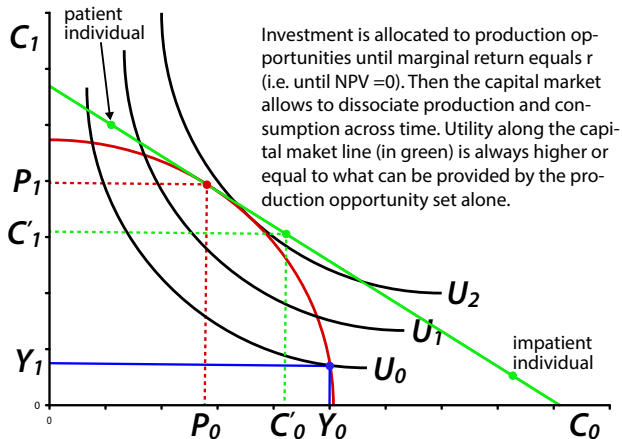
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You now have an opportunity to borrow (or lend) at the market determined rate r from (to) the capital market. The capital market allows to consume any combination of C_0 and C_1 along the capital market line (in green), from $W_0 = Y_0 + Y_1/(1+r)$ to $W_1 = Y_0(1+r) + Y_1$, and some combinations deliver utility above U_0 !

Both capital market and production opportunities exists

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All shareholders agree on which production opportunities to undertake (wealth and enterprise value is maximized once marginal return = the cost of capital r , i.e. invest until $NPV=0$ with production P_0 and P_1). This allows each investor to maximize its utility irrespective to its individual time preferences.

Fisher Separation Theorem: Implications

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Utility maximization requires wealth maximization.

- Fund projects until the expected NPV equals zero when using the appropriate discount rate;
- But which discount rate is appropriate? We will answer that later.

As long as management is diligent in its capital allocation and maximize enterprise value, there is no need for management to take into consideration the individual time preferences of shareholders since under the Fisher Separation Theorem each shareholder would have come to a decision which would have been identical to the decision of management (sometime referred to as the 'Unanimity Principle').

Once enterprise value (wealth) has been maximized, each shareholder can make its own decisions about what to consume when according to its time preferences, needs, and circumstances.

Critique of the Fisher Separation Theorem

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The model underlying the Fisher Separation Theorem relies upon a number of rather strong assumptions (notably perfect and complete capital markets). In addition, the model is more akin to a partial equilibrium/short term model than a long run general equilibrium model. So, the practical usefulness of the model (e.g. to make economic predictions) is probably limited.

But the logic illustrated by the model that investors can manage their consumption streams using the capital market once enterprise value (wealth) has been maximized through careful production/investment decisions is nevertheless very compelling.

In particular, it has been noted that the Theorem is not robust to the assumption of competition, and relaxing it likely nullifies the Unanimity Principle. However, it has been suggested that the NPV rule still ensures that management decisions would be Pareto-optimal (i.e. best solution for shareholders altogether, if not for each and every shareholder one at a time).

Textbook sections covered

- 4.1 to 4.8

Concept checks

- Chapter 4 of the textbook is kind of a long worked example illustrating the Fisher Separation Theorem.

Exercises

- Suggest solving 4.8 as a capital raising and return exercise.
 - ▶ Hint: a) $PV=AE$, $NPV=DE$; b) $CF/BD-1$; c) $AF/AB-1$