UNIVERSITY OF CALGARY
HASKAYNE SCHOOL OF BUSINESS

Corporate Finance

CAPM (empirical)

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Agenda

Experience the Efficient Frontier

CAPM

Estimate the Market Risk Premium

Estimate β of IBM

Estimate $E(R)$ for IBM using CAPM

Empirical tests of CAPM

Fama French three-factor model (FF3F)

Estimate FF3F for IBM

Estimate $E(R)$ for IBM using FF3F

Experience the Efficient Frontier

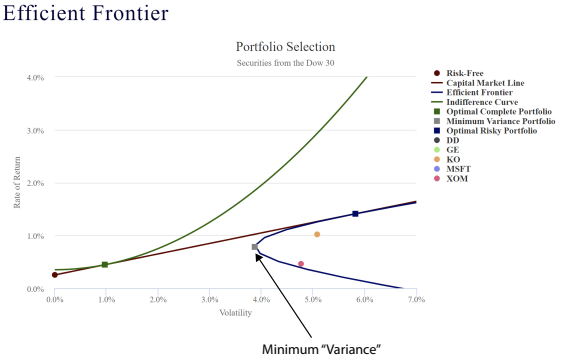
When you increase the number of securities in your portfolio of risky securities, you are likely to increase diversification, which improves your reward to risk universe, therefore expanding your efficient frontier from the south-east quadrant of the reward to risk plane toward the north-west quadrant. It then significantly increases the slope of the CML, leading investors to increase the proportion of risky assets in their portfolio. WDRS has a simulator to help you visualize such relationships.

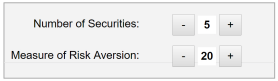
Login at WRDS (Wharton Research Data Services, to which HAskayne subscribes)

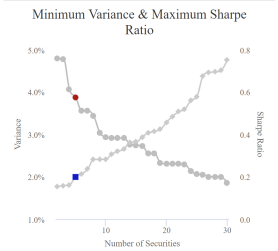
- Go to https://wrds-web.wharton.upenn.edu/wrds/
- Login (Username: fnce451s2020 Password: FNCE451S2020)
- Accept Terms of use (scroll down and accept)
- Click 'Classroom' and then click 'Investments' from the top bar menu
- Scroll down (fifth row left column) to 'Efficient Frontier', click on 'More>>' and then click on 'Link to Platform'

Experience the Efficient Frontier

Experiment with the controls, note your observations and discuss them with your group teammates.







Sharpe Ratio = $\frac{R_p - R_f}{\sigma_p}$ (slope of the CML, a reward to total risk ratio)

CAPM: a structural model

CAPM is a forward looking model

$$E\left(\widetilde{R}_i\right) = r_f + \beta_i \left[E\left(\widetilde{R}_m\right) - r_f\right]$$

- $\widetilde{R}_i$: return of security i (a random variable)
- $\widetilde{R}_m$: market return (a random variable)
- r_f: risk free rate of return
- β_i: beta of security i
- $E(\cdot)$: expectation operator (expected value of a random variable, a mean)

It is impossible to observe the future and quite difficult to measure expectations about the future, so a practical implementation of the model requires using some key assumptions.

- The distributions of $\widetilde{R}_i$ and $\widetilde{R}_m$ are iid (independent and identically distributed).
- The iid assumption allows using past realized returns in a forward looking manner (i.e. the distributions of $\widetilde{R}_i$ and $\widetilde{R}_m$ observed in past returns will continue unchanged in the future);
- β_i: beta of security i is constant through time.

Estimating CAPM for a given stock

CAPM: the estimation model

$$R_{i,t} - r_{f,t} = \alpha_i + \beta_i \left[R_{m,t} - r_{f,t}\right] + \epsilon_{i,t}$$

- $R_{i,t}$: return of security i in time period t
- $R_{m,t}$: market return in time period t
- $r_{f,t}$: risk free rate of return in time period t
- β_i: beta of security i (a parameter)
- α_i: excess return of security i (a parameter)
- $\epsilon_{i,t}$: error terms (a series of residual terms)

The data used is observations from $t = 1$ to $t = T$ for the three time series $R_{i,t}$, $R_{m,t}$, and $r_{f,t}$.

β_i and α_i are parameters to be estimated using an econometric technique (e.g. ordinary least square regression), thereafter denoted $\hat{\beta}_i$ and $\hat{\alpha}_i$.

Exercise 2: Estimate the Market Risk Premium

The market risk premium is the compensation for risk shareholders expect to receive for holding the market portfolio in excess of the risk free rate.

- It is typically expressed as a return above and over the risk-free rate.
- The simplest approach is to estimate it using an historical mean of the difference of the relevant observed returns (i.e. the observed market return less the risk free rate).

$$E\left(\widetilde{R}_m\right) - r_f \sim \overline{R_m - r_f} = \frac{1}{T}\sum_{t=1}^{T}\left[R_{m,t} - r_{f,t}\right]$$

Implementation

- Get monthly or daily returns for a broad equity index and for the risk-free rate for a long period.
- Calculate the average of the difference between the two time series.
- Verify if the mean is statistically significant (i.e. $\overline{R_m - r_f} \neq 0$).

Exercise 2: Estimate the Market Risk Premium

Get the Excel file 'CAPM_Empirical_RawData_V1' in D2L and open it.

Create a new variable in column I (=C2-B2) (i.e. market premium = market return — risk-free rate).

Calculate its arithmetic mean for the periods as below indicated.

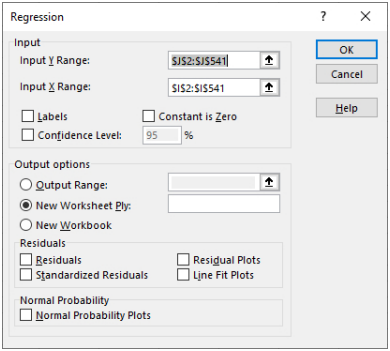
	Monthly	**p-value**	**Annualized**	**Rows**
1971 to 2015	0.27%	0.08	3.32%	I2:I541
1971 to 1985	-0.08%	0.60	-0.98%	I2:I181
1986 to 2000	0.68%	0.02	8.49%	I182:I361
2001 to 2015	0.22%	0.25	2.65%	I362:I541
1986 to 2015	0.45%	0.03	5.53%	I182:I541

Which period is 'reasonable' to choose? (statistical significance preferably requires p-value$<$0.05)

Exercise 3: Estimate the β of IBM (full sample)

Create a new variable in column J (=D2-B2) (excess return for IBM = IBM return — risk-free rate).

Regress J over I (>Data>Data Analysis>Regression).



Exercise 3: Estimate the β of IBM (full sample)

Output of the regression

Regression Statistics	
Multiple R	0.572576761
R Square	0.327844147
Adjusted R Sq	0.326594787
Standard Erro	0.058233432
Observations	540

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.889865769	0.889865769	262.4096042	2.32437E-48
Residual	538	1.82442935	0.003391133		
Total	539	2.714295118			

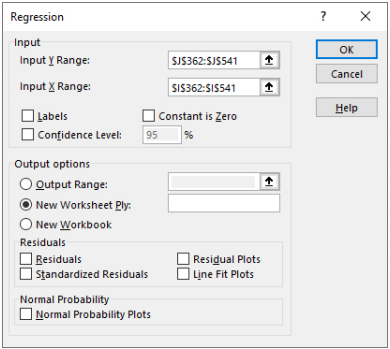
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.002089403	0.002510774	0.832174994	0.405679402	-0.00284272	0.007021524	-0.00284272	0.007021524
X Variable 1	0.923772282	0.057026283	16.19906183	2.32437E-48	0.811750812	1.035793753	0.811750812	1.035793753

The very high F-value indicates that the regression is statistically significant (good).
The high P-value of the intercept indicates that α is not statistically significant (good).
The P-value of zero for the slope indicates that β is highly significant (good).
The point estimate of β is 0.92 (from 0.81 to 1.04 in a +/- 5% confidence interval).

Exercise 4: Estimate the β of IBM (last 15 years)

Last 45 years is likely too far in time when estimating β.

Regress J over I for the last 15 years.



Exercise 4: Estimate the β of IBM (last 15 years)

Output of the regression

Regression Statistics	
Multiple R	0.600703324
R Square	0.360844483
Adjusted R Sq	0.357253722
Standard Erro	0.056751047
Observations	180

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.323654227	0.323654227	100.4924723	4.92343E-19
Residual	178	0.573281273	0.003220681		
Total	179	0.8969355			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.003068948	0.004235317	0.724608813	0.469643441	-0.00528895	0.011426842	-0.00528895	0.011426842
X Variable 1	0.978804695	0.097640339	10.02459337	4.92343E-19	0.786123116	1.171486274	0.786123116	1.171486274

The very high F-value indicates that the regression is statistically significant (good).
The high P-value of the intercept indicates that α is not statistically significant (good).
The P-value of zero for the slope indicates that β is highly significant (good).
The point estimate of β is 0.98 (from 0.79 to 1.17 in a +/- 5% CI).

Altogether: Estimate $E(R)$ for IBM using CAPM

$$E\left(\widetilde{R}_i\right) = r_f + \beta_i \left[E\left(\widetilde{R}_m\right) - r_f\right] \rightsquigarrow E(R_i) = r_f + \hat{\alpha}_i + \hat{\beta}_i \left[\overline{R_m - r_f}\right]$$

$$E(R_i) = 2.25\% + 0\% + 0.979 \times 5.53\% = 7.66\%$$

If IBM is unlevered (i.e. has no debt), an annual discount rate of 7.66% should be used when calculating the NPV of a project of the same risk as the firm.

N.B. Usually α_i is assumed to be zero.

Empirical tests of CAPM

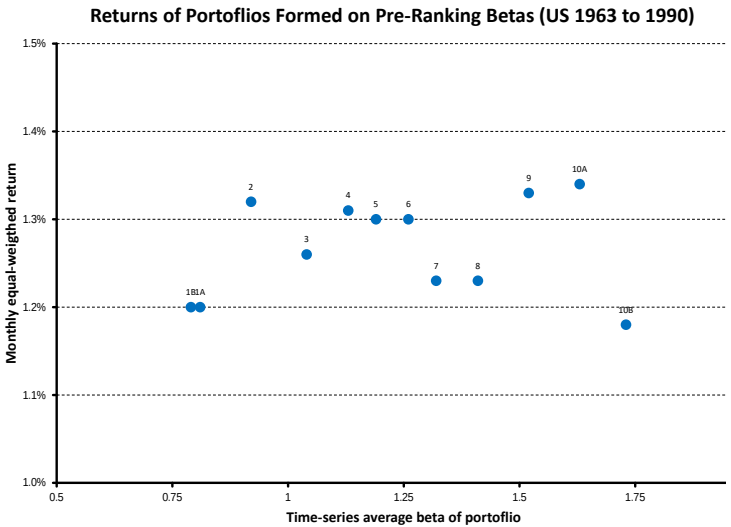
In science, any theory or hypothesis, regardless how mathematically sound or intuitively seductive, needs to be empirically tested.

- It is not possible to prove a theory, but it is possible to disprove it.
- When using various empirical tests, if all such tests fail to disprove a theory repeatedly (like it is the case with the theory of relativity of Einstein), such a theory gains scientific credibility.

CAPM has often been tested empirically and is now viewed as useful to learn and easy to use, but "inadequate for explaining the risk-return trade-off and the role that market risk plays in the determination of stocks' excess returns." (especially for individual stock returns)

- Initial studies used individual security returns and found intercepts with values much larger than the risk-free rate of return.
- This might result from statistical problems with individual betas, so use portfolios, but intercepts still too large and the SML too flat.
- Then other factors were found to influence security returns (incompatible with a single market factor CAPM).

Fama and French (1992) empirical test of CAPM



Fama French (1992) three-factor model (FF3F)

The Fama-French three factors are: the market risk, the risk of small versus big firms, and the risk of high book to market (growth) versus small book to market (value) firms.

$$E\left(\widetilde{R}_i\right) = r_f + \beta_{1i}\left[E\left(\widetilde{R}_m\right) - r_f\right] + \beta_{2i}\left[SMB\right] + \beta_{3i}\left[HML\right]$$

- $\widetilde{R}_i$: return of security i (a random variable)
- $\widetilde{R}_m$: market return (a random variable)
- r_f: risk free rate of return
- SMB : Small Minus Big, the small size premium
- HML : High Minus Low, the value premium
- β_i: beta of security i for each risk factor
- $E(\cdot)$: expectation operator (expected value of a random variable, a mean)

Fama French (1992) three-factor model (FF3F)

Small minus big factor (SMB).

- The size premium (SMB) is the average monthly return on the smallest 30% of stocks (in terms of market capitalization) minus the average monthly return on the largest 30%.
- When small stocks do well relative to large stocks, this will be positive; when they do worse than large stocks, this will be negative.

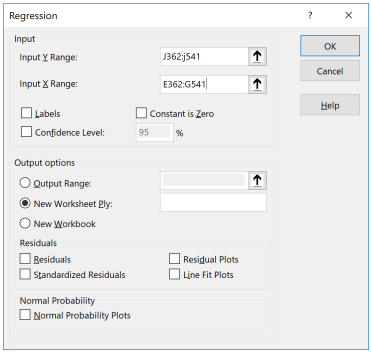
High minus low factor (HML)

- The value premium (HML) is the average monthly return for the 50% of stocks with the highest book-to-market ratio minus the average return for the 50% of stocks with the lowest book-to-market ratio.
- When high value stocks do well relative to low value stocks, this will be positive; when they do worse than low value stocks, this will be negative.
- High book-to-market stocks are considered "value" stocks; low book-to-market stocks are considered "growth" stocks.

A factor beta (such as above) is the sensitivity of security's returns to a given systematic risk factor.

Exercise 5: Est. β_1, β_2, & β_3 of IBM (last 15 years)

Estimate FF3F by regressing J (IBM excess return) over E:G (FF factors) for the last 15 years.



Exercise 5: Est. β_1, β_2, & β_3 of IBM (last 15 years)

Output of the regression

Regression Statistics	
Multiple R	0.6519329
R Square	0.425016507
Adjusted R Sq	0.415215651
Standard Erro	0.054131737
Observations	180

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	0.381212393	0.127070798	43.36524787	4.93947E-21
Residual	176	0.515723107	0.002930245		
Total	179	0.8969355			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	0.002223282	0.004079611	0.544973946	0.586461343	-0.00582797	0.010274534	-0.00582797	0.010274534
X Variable 1	0.987094221	0.095551829	10.33045863	7.44986E-20	0.7985194	1.175669042	0.7985194	1.175669042
X Variable 2	-0.08223629	0.166285618	-0.4945484	0.62153568	-0.41040667	0.245934099	-0.41040667	0.245934099
X Variable 3	-0.66971781	0.147103408	-4.5527008	9.85352E-06	-0.96003144	-0.37940417	-0.96003144	-0.37940417

The high F-value indicates that the regression is statistically significant (good).
The high P-value of the intercept indicates that α is not statistically significant (good).
The zero P-value of $X_1 \rightarrow \beta_1$ is highly significant (good and ~same as CAPM).
The high P-value of $X_2 \rightarrow \beta_2$ is not statistically significant (SMB no longer works).
The zero P-value of $X_3 \rightarrow \beta_3$ is highly significant (good).

Altogether: Estimate $E(R)$ for IBM using FF3F

$$E(R_i) = r_f + \hat{\alpha}_i + \beta_{1i}\,[MRK] + \beta_{2i}\,[SMB] + \beta_{3i}\,[HML]$$

$$E(R_i) = 2.25\% + 0\% + 0.987 \times 7.91\% + 0\% - 0.67 \times 2.52\% = 8.37\%$$

If IBM is unlevered (i.e. has no debt), an annual discount rate of 8.37% (versus a 7.66% CAPM rate) should be used when calculating the NPV of a project of the same risk as the firm.

N.B. Usually α_i is assumed to be zero.